

Regime Shift Warnings in the Lake Eutrophication Ecosystem Subject to Non-Gaussian Input Fluctuation

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Abstract

The lake eutrophication dynamics of bistable states in stochastic models have been investigated extensively in the regime of Gaussian perturbations, and fluctuations in environmental parameters can cause state switching between oligotrophic and eutrophic states, thereby creating a disruption of ecological balance. In this paper, we explore the impact of non-Gaussian fluctuations in phosphorus intake on the dynamic behavior of the bistable lake eutrophication ecosystem with phosphorus cycle. The bifurcation diagrams of the lake eutrophication ecosystem within bistability are described to show the non-Gaussian effect. It is found that the region of bistability is not only associated with the non-Gaussian noise intensity (α) but also with the correlation time (τ), which is the exhibition of non-Gaussian benefits. More interestingly, in the absence of fluctuations in recycling of phosphorus ecological bistability also appears in the range of the recycling parameter $r < 0.5$. Furthermore, we also analyze the effective potential under different parameters of non-Gaussian input noise and discuss the results of non-Gaussian noise-induced oligotrophication in detail. The above clues are beneficial to control regime shifts under the implications for the real lake eutrophication ecosystems.

Keywords

Non-Gaussian noise, Noise-enhanced stability, Lake eutrophication ecosystem, Phosphorus cycle.

1. Introduction

With the accelerating of urbanization and industrialization, the ecological environment problems have received more and more attentions in human life and production. The excessive use of nature resources not only destroy the ecological balance but also makes us vulnerable to catastrophic regime shift risks. Bistability, such as desert and vegetational states [1], extinct and tumorigenic states [2], and oligotrophic and eutrophic states [3], exists widely in the ecological systems with dramatically dynamic behaviors. For instance, ecosystems sometimes encounter a sudden regime shift from one stable state to another distinctly different stable state, this process may result in the degradation of ecosystem services as well as the consequent economic losses. Regime shifts in ecological systems are filled with fascinating dynamic behavior, which inspires us to pursue the ecological mechanism behind encouraging dynamics

and hysteresis in ecosystems. Since the regime shifts are usually nonreversing within the hysteresis effect, it is of great concern in ecological resource management and also in maintaining ecosystem services for human beings.

Previous ecological dynamic analysis approaches generally assume the ecological systems are deterministic (i.e., static ecological models) [4]-[7]. However, many real ecosystems often have intrinsic stochasticity related to the ceaselessly changing and interfering ecology participants (i.e., external environment induced fluctuations), which is able to control the status switching between the two stable states straightly [8]-[10]. More interestingly, the sudden steady-state transition is excited by gradually changing external factors instead of huge exogenous shocks, which can be recognized as catastrophic regime shifts. Over the past decade, environmental disturbance plays an important role in ecosystem modeling, which may be useful in dynamics and resilience of ecological systems [11]-[13]. For example, Wang et al. investigated the role of environmental noises in a mean-field vegetation model, they observed the resonant phenomenon induced by the interaction between noises using the mean first-passage time (MFPT) index [11]. The perturbations of plankton ecosystems are explored in [12] and [13] to control system stabilisation. Besides, the investigation of noise-enhanced stability (NES) in physical and biological systems has been widely investigated [14]-[18].

Lake ecosystems generally exhibit two preferential states with typical bistability, while low and high phosphorus regimes are recognised respectively as oligotrophic and eutrophic lakes. The existing lake bistability is commonly related to the phosphorus recovery of sediments. van Nes et al. demonstrated rich bifurcation characteristics induced by the environmental perturbations, and the region of bistability is associated with the alternative attractors [19]. Zeng et al. considered the random fluctuations with correlation to environmental parameter in lake approaching eutrophication ecosystems, they showed that correlated noises can dramatically enlarge hysteresis loop acreage [20]. In addition, a rising standard deviation indicator is introduced into lake eutrophication ecosystems that are easy to forecast an beforehand catastrophic regime shift [21]. In practice, frequent regime shifts are more likely to occur in real ecosystems, and it is consistent with the theoretical model predicted. After that, Wang et al. explored phosphorus dynamics under large external impacts and revealed the flickering gives early warning information before the catastrophic eutrophic transition [22]. Recently, Zeng et al. discovered that the input and loss noises can serve as controllable parameters over regime shifts of oligotrophication and eutrophication in a time-delayed lake ecosystem [23].

In spite of the above mentioned successful applications of Gaussian noise-induced regime shifts in lake ecosystems, the correspondingly dynamic behavior of non-Gaussian perturbation case remains a key challenge. The benefit of non-Gaussian effect has been extensively studied in nonlinear systems, such as biological, physical and neural systems, through theoretical and experimental discussion [24]-[29]. It is hard to solve stochastic differential equations with non-Gaussian term due to the missing Markov property, and thus different effective methods including numerical simulation and theoretical approximation have always fascinated researchers. Some literatures focus on non-Gaussian effect in life science fields. For example, Guo et al. explored the dynamical behavior of non-Gaussian Lévy sources under the FitzHugh-Nagumo model, which exploits the usefulness of the Janicki-Weron and Runge-Kutta algorithms in NES and resonance activation problems [30]. Afterwards, Dong et al. investigated dynamics in a foraging colony model in the case of non-Gaussian involves. They showed in the stationary probability distribution the non-Gaussian noise can control the steady state stability and cause peak transition between single and double peaks [31]. Moreover, dynamical features in ecological systems and non-Gaussian induced ecosystems stable have been investigated [32]-[36].

To the best knowledge of the authors, there are no empirical evidence to explore the dynamics behavior of non-Gaussian effect in lake ecosystems. Especially, the differences between non-Gaussian and Gaussian perturbations on lake ecosystems seem to be particularly important, which is beneficial to make a priority management strategy for lake environment protection. In this paper, we aim to investigate the dynamics of a lake eutrophication model that includes non-Gaussian input noise. The hysteresis loop acreage can be enlarged by the non-Gaussian input noise in comparison with the deterministic case, which is similar to that in literature [20]. Meanwhile, the effective potential function is utilized to explore the regime shifts between oligotrophication and eutrophication induced by non-Gaussian noises.

2. The Non-Gaussian Additive Noise Induced Regime Shifts

Phosphorus intake is mainly composed of the load from the watershed and recyclable at the sediment, while phosphorus output involves the deposition, efflux and sequestration of the consumer or benthic biomass. The lake eutrophication model that characterizes phosphorus cycle in lakes without external perturbation can be written as

$$\frac{dp}{dt} = c - mp + rs(p) \quad (1)$$

Here, the parameter c is the rate of phosphorus intake in the watershed, p is phosphorus concentration in the lake water, and m is the loss rate of phosphorus. The parameter r is the recycling coefficient in concern with hypolimnion being anoxic, which denotes the maximum rate of recyclable phosphorus. The term $s(p)$ is the known Hill function that induces two alternative regimes, i.e., ecological bistability. The ecological bistability consists of a high phosphorus concentration that recognizes as the eutrophic state and a low phosphorus concentration that equals in the oligotrophic state, which is mainly controlled by the recycling coefficient r .

We begin with the stability behavior study for the phosphorus concentration in the deterministic lake ecosystem, only one equilibrium point (c/m) can be achieved in the case of $r = 0$. Besides, it is recalled that the Hill term $s(p)$ can induce two alternative stable states, which is increased rapidly on the threshold w :

$$s(p) = \frac{p^w}{z^w + p^w} \quad (2)$$

On account of environmental perturbations in phosphorus intake such as dry deposition, rainfall and groundwater, non-Gaussian noise inevitably participates in the input of phosphorus. Thus, we explore the environmental perturbation's effect on the ecological bistability with the term $c + \mathcal{G}(t)$ in the initial Langevin equation.

According to the above mentioned non-Gaussian noise $\mathcal{G}(t)$ and Hill function $s(p)$ with the initial parameters $m = 1$, $z = 1$ and $w = 8$, we can rewrite the Langevin equation corresponding to Eq. (1) as

$$\frac{dp}{dt} = c - p + \frac{rp^8}{1 + p^8} + \mathcal{G}(t) \quad (3)$$

In Eq. (4), the non-Gaussian term $\mathcal{G}(t)$ can be expressed as

$$\frac{d\mathcal{G}(t)}{dt} = -\frac{1}{\tau} \frac{d}{d\mathcal{G}} \gamma_q(\mathcal{G}) + \frac{1}{\tau} \xi(t) \quad (4)$$

with

$$\gamma_q(\mathcal{G}) = \frac{\alpha}{\tau(q-1)} \ln \left[1 + \frac{\tau}{\alpha} (q-1) \frac{\mathcal{G}}{2} \right] \quad (5)$$

where $\xi(t)$ is Gaussian white noise with the following statistical properties

$$\gamma_q(\mathcal{G}) = \frac{\alpha}{\tau(q-1)} \ln \left[1 + \frac{\tau}{\alpha} (q-1) \frac{\mathcal{G}}{2} \right] \quad (6)$$

α is the intensity of $\xi(t)$, the first and second moments of non-Gaussian noise $\mathcal{G}(t)$ are

$$\langle \mathcal{G}(t) \rangle = 0, \quad \langle \mathcal{G}^2(t) \rangle = \begin{cases} \frac{2\alpha}{\tau(5-3q)}, & q \in \left(-\infty, \frac{5}{3} \right) \\ \infty, & q \in \left[\frac{5}{3}, 3 \right) \end{cases} \quad (7)$$

Here, we use the parameter q to exhibit the departure from Gaussian noise and τ to show the correlation time of the non-Gaussian noise $\mathcal{G}(t)$.

Based on the path integral method, in the case of $|q-1| \ll 1$, we have

$$\begin{aligned} \frac{1}{\tau} \frac{d}{d\mathcal{G}} \gamma_q(\mathcal{G}) &= \frac{\mathcal{G}}{\tau \left[1 + \frac{\mathcal{G}}{\alpha} (q-1) \frac{\mathcal{G}^2}{2} \right]} \\ &\approx \frac{\mathcal{G}}{\tau \left[1 + \frac{\mathcal{G}}{\alpha} (q-1) \frac{\langle \mathcal{G} \rangle^2}{2} \right]} \\ &= \frac{\mathcal{G}(t)}{\tau_{eff}} \end{aligned} \quad (8)$$

Substitute the Eq. (8) into Eq. (4), we can rewrite the non-Gaussian white noise as:

$$\frac{d\mathcal{G}(t)}{dt} = -\frac{1}{\tau_{eff}} \mathcal{G}(t) + \frac{1}{\tau_{eff}} \Gamma(t) \quad (9)$$

Here, $\Gamma(t)$ is the additive Gaussian noise with the statistic characteristics

$$\langle \Gamma(t) \rangle = 0 \quad (10)$$

$$\langle \xi(t) \xi(t') \rangle = 2\alpha_{eff} \delta(t-t') \quad (11)$$

Where τ_{eff} and α_{eff} denote the effective correlation time and the effective noise intensity, respectively

$$\tau_{eff} = \frac{2(2-q)}{5-3q} \tau \quad (12)$$

$$\alpha_{eff} = \left[\frac{2(2-q)}{5-3q} \right]^2 \alpha \quad (13)$$

If the limit of $q \rightarrow 1$, the non-Gaussian noise $\mathcal{G}(t)$ can be approximated as colored Gaussian noise with correlation time τ_{eff} and noise intensity α_{eff} . In the case of $q \neq 1$, $\mathcal{G}(t)$ denotes the non-Gaussian noise term.

By using the unified colored noise approximation method [37], the non-Markov of the Langevin Eq. (1) can be approximated as one-dimensional Markov one

$$\frac{dp}{dt} = \tilde{h}(p) + \tilde{g}(p) \tilde{\Gamma}(t) \quad (14)$$

where

$$\tilde{h}(p) = \frac{h(p)}{\mathbb{C}(\tau_{eff}, p)} \quad (15)$$

$$\tilde{g}(p) = \frac{\sqrt{\alpha_{eff}}}{\mathbb{C}(\tau_{eff}, p)} \quad (16)$$

and

$$\langle \tilde{\Gamma}(t) \tilde{\Gamma}(t') \rangle = 2\delta(t-t') \quad (17)$$

$$\mathbb{C}(\tau_{eff}, p) = 1 - \tau_{eff} h'(p) \quad (18)$$

Thus, the approximate Fokker-Planck equation corresponding to Eq. (1) with Eq. (2) is derived by using the Novikov theorem and the Fox approach [38]. It is as follows:

$$\frac{\partial}{\partial t} Q(p, t) = L_{FP} Q(p, t) \quad (19)$$

$$L_{FP} = -\frac{\partial}{\partial p} A(p) + \frac{\partial^2}{\partial p^2} B(p) \quad (20)$$

with

$$A(p) = \tilde{h}(p) + \tilde{g}(p)\tilde{g}'(p) \quad (21)$$

$$B(p) = \tilde{g}^2(p) \quad (22)$$

It is noted that Eqs. (19) and (20) are continuity equations of the probability distribution, $Q(p, t)$ merely represents the probability of the current moment. In the case of $t \rightarrow +\infty$, $Q(p, t)$ vanishes at the boundary. According to this situation, $Q(p, t)$ is a time-independent stochastic processes, i.e., $\frac{\partial Q(p, t)}{\partial t} = 0$. Consequently, considering the natural boundary conditions in Eqs. (19) and (20), the stationary probability distribution $Q_s(p)$ can be obtained by the following solution of the differential equation

$$-\frac{d}{dp}[A(p)Q_s(p)] + \frac{d^2}{dp^2}[B(p)Q_s(p)] = 0 \quad (23)$$

By integrating with respect to p in Eq. (23), we rewrite it as

$$-A(p)Q_s(p) + \frac{d}{dp}[B(p)Q_s(p)] = R \quad (24)$$

where the right-hand side parameter R denotes an integration constant independent of p . It is notably that $Q_s(p) = 0$ with p being the region at infinity, and obtain $R = 0$.

Then we solve the stationary probability distribution function $Q_s(p)$ corresponding to Eq. (24), and yield

$$\begin{aligned} Q_s(p) &= \frac{N}{B(p)} e^{\int^p \frac{A(p)}{B(p)} dp} \\ &= \frac{N}{\tilde{g}(p)} e^{\int^p \frac{\tilde{h}(p)}{\tilde{g}^2(p)} dp} \\ &= N e^{-\phi(p)} \end{aligned} \quad (25)$$

where the effective potential $\phi(p)$ reads

$$\begin{aligned} \phi(p) &= \ln \tilde{g}^2(p) - \int^p \frac{A(p)}{\tilde{g}^2(p)} dp \\ &= \ln \mathbf{W} + \frac{p}{2\alpha_{eff}}(p - 2c - 2r)\tau_{eff} + \frac{(p + p^9 + r)[(p - 2c)(1 + p^8) - (1 + 2p^8)r]}{2\alpha_{eff}(1 + p^8)^2} \\ &\quad + \frac{r\tau_{eff}}{8\alpha_{eff}} \cos(\pi/8)(2\mathbf{E}_1 + 2\mathbf{E}_2 + \mathbf{K}_1) - \frac{r\tau_{eff}}{8\alpha_{eff}} \sin(\pi/8)(2\mathbf{F}_1 - 2\mathbf{F}_2 - \mathbf{K}_2) \end{aligned} \quad (26)$$

With

$$\mathbf{W} = \frac{\sqrt{\alpha_{eff}}}{1 + \tau_{eff} - \tau_{eff} \frac{8rp^7}{1 + p^8} + \tau_{eff} \frac{8rp^{15}}{(1 + p^8)^2}} \quad (27)$$

$$\mathbf{E}_1 = \arctan \left[p \sec(\pi/8) - \tan(\pi/8) \right] \quad (28)$$

$$\mathbf{E}_2 = \arctan \left[p \sec(\pi/8) + \tan(\pi/8) \right] \quad (29)$$

$$\mathbf{K}_1 = \ln \frac{1 + p^2 + 2p \cos(\pi/8)}{1 + p^2 - 2p \cos(\pi/8)} \quad (30)$$

$$\mathbf{F}_1 = \arctan \left[\cot(\pi/8) - p \csc(\pi/8) \right] \quad (31)$$

$$\mathbf{F}_2 = \arctan \left[\cot(\pi/8) + p \csc(\pi/8) \right] \quad (32)$$

$$\mathbf{K}_2 = \ln \frac{1 + p^2 + 2p \sin(\pi/8)}{1 + p^2 - 2p \sin(\pi/8)} \quad (33)$$

In the following, we demonstrate with regime shifts that this non-Gaussian input noise alone indeed induced ecological bistability.

3. Dynamic Behaviors of Regime Shift under Different Non-Gaussian Input Noise Parameters

In the previous lake eutrophication model introduced by Zeng et al [20], they dedicate to the study the cross-correlation relationship between the input and recycling perturbations with Gaussian white noises over the lake system, in the mind-provoking works. Here, we will investigate the effect of the non-Gaussian additive input noise alone on the dynamic behavior in inducing regime shifts.

In Figure 1, we draw the S shape curves for the rate of phosphorus intake c as a function of phosphorus concentration p in the deterministic model, and explore the ecological bistability induced by the different recycling parameter r . Similar to the literature [20], we set the fixed parameters as $m=1$, $z=1$ and $w=8$ to exhibit the variation of the region of ecological bistability. It is observed that the ecological bistability vanishes in the relatively low recycling coefficient, i.e., $r < 0.5$. Besides, when the recycling coefficient exceeds 0.5 the ecological bistability phenomenon can be awakened. What is more, the region of ecological bistability expands with the increasing of recycling coefficient. Therefore, we can obtain the existence of ecological bistability for the deterministic model in the absence of environmental fluctuations as $r > 0.5$.

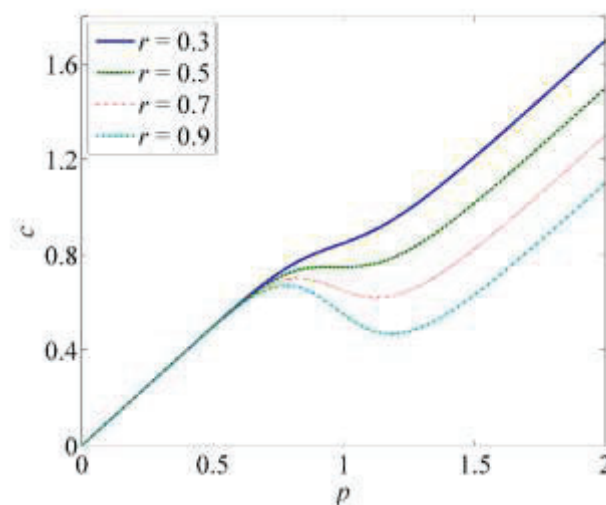


Figure 1. The S shape bifurcation diagram in the deterministic lake model for the rate of phosphorus intake c as a function of the phosphorus concentration p under different recycling coefficient r .

Since the rate of phosphorus intake in the watershed continually fluctuates driven by the non-Gaussian noise, we employ its non-Gaussian characteristic parameters to classify the S shape bifurcation in the stochastic lake model: there are plotted along the ordinate in Figure 2 and Figure 3. In Figure 2, The S shape bifurcation diagram for the rate of phosphorus intake c as a function of the phosphorus concentration p under different non-Gaussian input noise intensity is shown to exhibit the non-Gaussian effect in inducing regime shifts between oligotrophic state and eutrophic state. When the stochastic lake model suffer from non-Gaussian input noise, it can induce ecological bistability. The hysteresis loop acreage will extend with the increscent of noise intensity, which is directly changes the bifurcation diagram. It needs to be noticed that the stochastic lake model subject to non-Gaussian input noise alone would like to switch from the eutrophic state to the oligotrophic state, we should explore the dynamic behavior to provide a control strategy. Besides, it is interesting to note that non-Gaussian input noise alone can cause the ecological bistability phenomenon even in the area of $r < 0.5$.

Figure 3 shows the S shape bifurcation diagram for the rate of phosphorus intake c as a function of the phosphorus concentration p under different correlation time of the non-Gaussian noise τ , the ecological bistability induced by the correlation time will be investigated. It is also observed that the hysteresis loop acreage enlarge with the increasing correlation time, which is a reflection of non-Gaussian input noise lead to ecological bistability. In the case of small correlation time ($\tau = 0.01$), the ecological bistability does not exist. As the correlation time τ is increased ($\tau = 0.05, \tau = 0.09$), the stochastic lake model varies from a single high-concentration (phosphorus) nutrient state to a bistable region with alternative stable states. In addition, it can be seen that ecological bistability still exists in spit of $r < 0.5$.

According to the above description of bifurcation diagrams, we can conclude that the non-Gaussian input noise caused the ecological bistability, which is a direct result of the expanding hysteresis loop area in the stochastic lake model. What makes sense is to arouse the ecological bistability phenomenon in the condition of $r < 0.5$, the result of our paper is similar to the cross-correlation between input and recycling noises induced bistability in the stochastic lake model [20]. Recall that the two noises with correlation can break through the restriction of $r > 0.5$, we also found that the non-Gaussian input noise can cause bistability. Two distinguishing features that occur owing to the non-Gaussian input noise: the oligotrophic state and the eutrophic state. Due to the bistability, lake is at risk of shifting from oligotrophication to eutrophication, besides

it is also acquires an opportunity to facilitate ecological restoration from eutrophic state under certain conditions. In view of this, we are dedicated to discuss how non-Gaussian input noise induced regime shifts from eutrophication to oligotrophication.

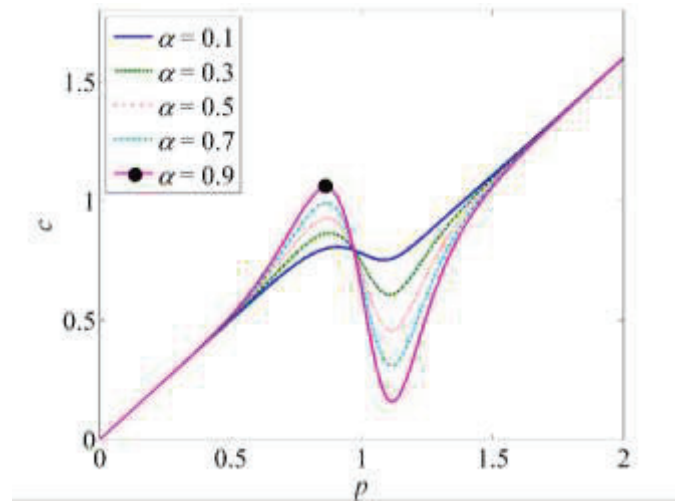


Figure 2. The S shape bifurcation diagram in the stochastic lake model for the rate of phosphorus intake c as a function of the phosphorus concentration p under different non-Gaussian input noise intensity α , the other parameters are $q = 0.9$, $\tau = 0.08$, $r = 0.4$.

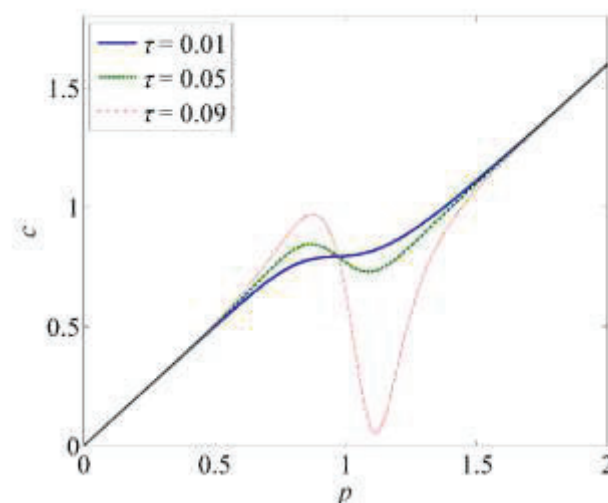


Figure 3. The S shape bifurcation diagram in the stochastic lake model for the rate of phosphorus intake c as a function of the phosphorus concentration p under different correlation time of the non-Gaussian noise τ , the other parameters are $q = 0.9$, $\alpha = 0.5$, $r = 0.4$.

To demonstrate that the lake ecosystem maintaining in the desired desirable (oligotrophic) state with the help of input noise, we provide the graphic descriptions of effective potential function in which the nutritional status of phosphorus can be switched using the controlling parameter of non-Gaussian input noise in the lake eutrophication ecosystem. From a prephysics perspective, the dynamics behavior of the phosphorus concentration in the lake eutrophication ecosystem mainly depend on the generalized force. Besides, it is well-known that the generalized force can be expressed as the negative derivative of the effective potential over the stochastic lake model. The local minimum on the left and right in the effective potential denote

the two stable states (p_{sL} and p_{sR}) and the maximum one denotes the unstable state (p_{s0}), while the left and right stable states induced the low nutrient phosphorus and high nutrient phosphorus, respectively, and the unstable state exhibit the valley in phosphorus probability. Here, non-Gaussian input noise-induced regime shifts from the eutrophic state to the oligotrophic state may occur. In the field of physics, we refer to the valley of $\phi(p)$ associated with p_{sL} (or p_{sR}) as potential well and the peak of $\phi(p)$ associated with p_{s0} as potential barrier. Without loss of generality, we regard the potential well corresponding to p_{sL} as the left well and the potential well corresponding to p_{sR} as the right well. Moreover, the depth of the left well can be expressed as $d_L = \phi(p_{s0}) - \phi(p_{sL})$, while the depth of the right well can be written as $d_R = \phi(p_{s0}) - \phi(p_{sR})$.

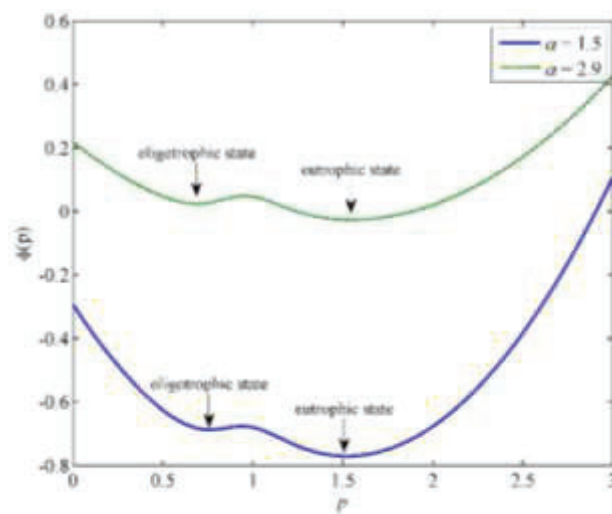


Figure 4. The effective potential as a function of the phosphorus concentration p in the stochastic lake model for the fixed rate of phosphorus intake $c = 1.1$ under different noise intensity of the non-Gaussian noise α , the other parameters are $q = 0.9$, $\tau = 0.08$, $r = 0.4$.

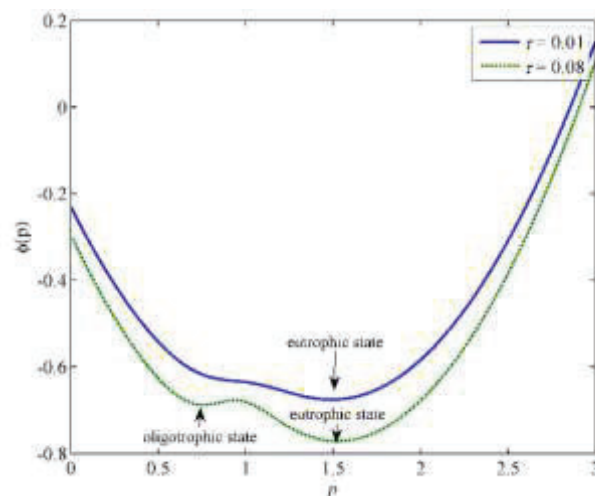


Figure 5. The effective potential as a function of the phosphorus concentration p in the stochastic lake model for the fixed rate of phosphorus intake $c = 1.1$ under different correlation time of the non-Gaussian noise τ , the other parameters are $q = 0.9$, $\alpha = 1.5$, $r = 0.4$.

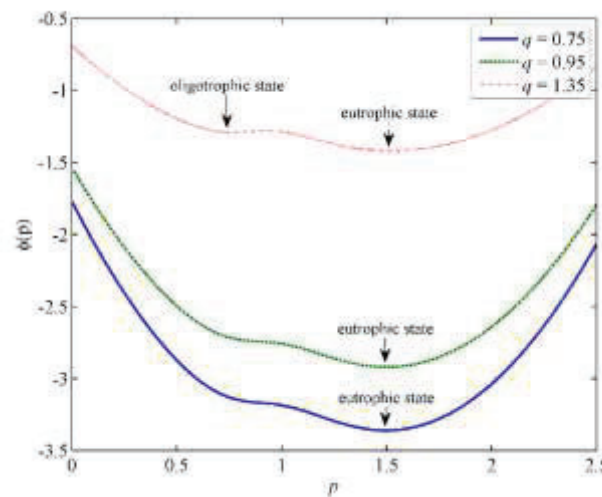


Figure 6. The effective potential as a function of the phosphorus concentration p in the stochastic lake model for the fixed rate of phosphorus intake $c = 1.1$ under different departure from Gaussian noise q , the other parameters are $\tau = 0.08$, $\alpha = 0.5$, $r = 0.4$.

In Figure 4, we investigate the effects of the noise intensity of the non-Gaussian noise on the noise-induced oligotrophication. Figure 4 exhibits the variant trajectories of the effective potential of $\phi(p)$, which switch randomly between the oligotrophic equilibrium point p_{sL} and the eutrophic equilibrium point p_{sR} . The effective potential shows a double valley under different noise intensities, which gives the opportunity to induce regime shifts. The depth of right well is greater than the depth of left well increases for low values of noise intensity ($\alpha = 1.5$), then its oligotrophication becomes more pronounced as the values of α increase to relatively large ($\alpha = 2.9$). It means that the increasing α may lead to a regime shift towards the oligotrophic state. Hence, it is remarkable that large enough non-Gaussian input noise intensity is needed to make oligotrophic state more dominant.

Next, the effects of the correlation time of the non-Gaussian noise τ on the noise-induced oligotrophication are also investigated. In Figure 5, the shape and depth of the potential well can be modified by the non-Gaussian noise parameter with correlation time. Interestingly, the effective potential concentrated in the right well alone for low values of correlation time ($\tau = 0.01$), while the left and right potential wells are both emergent for large values of correlation time ($\tau = 0.08$). In fact, real lake ecosystems may suffer from non-Gaussian disturbances with large correlation time in phosphorus intake from the watershed, which accelerates the regime shift to the eutrophic state.

Figure 6 gives the effects of the departure from Gaussian noise q on the noise-induced oligotrophication. Particularly, the effective potential manifests single valley feature in the right well for low values of the departure from Gaussian noise ($q = 0.75$, $q = 0.95$), which corresponds to the eutrophic equilibrium point p_{sR} . With the increasing of q ($q = 1.35$), the depth of right well shows a downward trend and the depth of left well begins to appear. Furthermore, the equilibrium state be more attracted to the left well (p_{sL}) with the increments in α , τ and q , which demonstrates that the non-Gaussian input noise alone can contribute to oligotrophication state in the stochastic lake ecosystem.

In general, the most effective way for ensuring the lake ecosystem remain positioned around a oligotrophic state is to weaken or eliminate the alternative eutrophic state. The parameter modulation of the non-Gaussian input noise may induce the regime shifts from the eutrophic state to the oligotrophic one, and thereby resulting in a controllable lake ecological restoration.

Thus, the investigation of the effective potential in the stochastic lake model is a good way to illustrate the mechanisms of the non-Gaussian input noise alone can also cause ecology restoration from eutrophication to oligotrophication.

4. Conclusion

If In Section 3, the role of non-Gaussian input noise alone on regime shifts in the stochastic lake model is examined to noise-induced oligotrophication effect for a new perspective. It is interesting that the lake eutrophication ecosystem can exhibit the ecological bistability within the non-Gaussian input noise effect. In the absence of fluctuations in recycling of phosphorus, non-Gaussian can also lead to regime shifts from an equilibrium state to the alternative one, even in the range of the recycling parameter $r < 0.5$. The implementation of the stochastic lake model with a single non-Gaussian input noise, without considering the recycling noise, can suppress the eutrophication by inducing the appearance of the left potential well. The increasing parameters of non-Gaussian input noise (α , τ , q) can not only arouse the ecological bistability but also stabilize the oligotrophic state, which demonstrates that this method can promote the ability of non-Gaussian noise-induced oligotrophication.

In this paper, we only provide the theoretical basis results with non-Gaussian noise-induced oligotrophication in the stochastic lake model. Since our research mainly concentrates on the effective potential function and only the paradigmatic lake eutrophication model is considered, it would be attractive to investigate numerical results using various evaluation indicators (such as mean first-passage time, residence probability, and amplitude difference, etc., [39]), or be extended to other stochastic complex systems (such as vegetation system, tumour system, metapopulation system, etc., [40]). Even this preliminary exploration achieves some valuable conclusions, it still exists open question to explore the ecological dynamics in forthcoming work.

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